WARM UP

Taieri Mouth inlet is tidal and the water rises and falls over the course of the day. The depth of the water is measured at certain points using permanently set poles.

- There is 6.25 hours between low and high tide.
- The depth at high tide is 4.5 m.
- The depth of water at low tide is 2 m.
- A research student begins her measurement readings of the tide on Saturday 7am. The depth is at its greatest at 8am.

a) Write the eqn for the tide at Taieri Mouth on that day.

A = (4.5 - 2)÷2=1.25

T = 6.25 x 2 = 12.5 hours

B = $\frac{2\pi}{12.5} = \frac{4\pi}{25}$

C = x – shift = -1 since the max value (greatest depth) occurs 1 hour after time zero/initial time which is 7am. Since we are shifting our graph to the right by 1, the C value is negative 1.

D = halfway between 4.5 and 2 = (4.5 + 2)÷2 = 3.25

Since the max value occurs near the starting time, we will choose the cos function:

$$y = 1.25\cos[\frac{4\pi}{25}(t-1)] + 3.25$$

b) What will the depth be at 12 noon that day?

Sub $t = 5$(i.e. 5 hours after 7am) into $y = 1.25\cos[\frac{4\pi}{25}(t-1)] + 3.25$

⇨ $y = 1.25\cos[\frac{4\pi}{25}(5-1)] + 3.25 = 2.718\ m.$

c) When is the depth first measured at 3m?

Solve $y = 3$ => solve $1.25\cos[\frac{4\pi}{25}(5-1)] + 3.25 = 3.$

Solve on GC, graph both $y_1 = 1.25\cos[\frac{4\pi}{25}(5-1)] + 3.25$ and $y_2 = 3.$

Solution: $t = 4.526 =$ 4 hours and 32 minutes after 7am. OR 11:32am.

PROVING TRIG IDENTITIES

Using: $\operatorname{cosec} x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\tan x = \frac{\sin x}{\cos x}$,

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ and $\cos^2 x + \sin^2 x = 1$.

Examples: Prove that

1) $\cos x \tan x \operatorname{cosec} x = 1$

LHS = $\cos x \tan x \operatorname{cosec} x = \cos x \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{\cos x \cdot \sin x}{\cos x \cdot \sin x} = 1$ = RHS.

2) $\sin\theta \tan\theta + \cos\theta = \sec\theta$

LHS = $\sin\theta \tan\theta + \cos\theta = \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta = \frac{\sin^2\theta}{\cos\theta} + \cos\theta = \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$ = RHS.

3) $\frac{\sec x}{\tan x} = \operatorname{cosec} x$

LHS = $\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos x} \div \frac{\sin x}{\cos x} = \frac{1}{\cos x} \times \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \operatorname{cosec} x$ = RHS.

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